PVP 14

Code: EE4T1

II B.Tech - II Semester–Regular/Supplementary Examinations–April 2018

COMPLEX VARIABLES & SPECIAL FUNCTIONS
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

PART – A

Answer ***all*** the questions. All questions carry equal marks

11 x 2 = 22M

1. a) Examine the nature of analyticity of $f(z) = z + 2\bar{z}$.

   b) Show that the function $u(x, y) = x^2 - y^2$ is harmonic.

   c) Find the real and imaginary parts of $\cosh z$.

   d) Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along the path $y = x$.

   e) Evaluate $\int_C (z - a)^{-1}\, dz$, where C is a simple closed curve and the point $z = a$ is inside C.

   f) Find the nature and location of singularity of $ze^{\frac{1}{z^2}}$.

   g) Determine the residue of $f(z) = \frac{\cos z}{z^2 + 2z}$ at the pole lie inside the circle $|z| = 1$.

   h) State residue theorem.

   i) Find the critical points of the transformation $w = \frac{2i - 6z}{iz - 3}$

   j) Find the inverse mapping of bilinear transformation.

   k) Find first two Legendre polynomials.

PART – B

Answer any ***THREE*** questions. All questions carry equal marks.

3 x 16 = 48 M

2. a) Find the analytic function $f(z) = u + iv$, if $u - v = \frac{x - y}{x^2 + 4xy + y^2}$. 8 M

b) If $f(z)$ is an analytic function of z, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}\right) |f(z)|^2 = 4 |f'(z)|^2$. 8 M

3. a) State and prove Cauchy's theorem. 8 M

b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region 8 M

(i) $1 < |z| < 2$ and (ii) $|z| > 2$.

4. a) Evaluate $\int_C \frac{\sin \pi z^2}{(z-1)^2 (z-2)} dz$, where $C : |z| = 3$. 8 M

b) Apply the calculus of residues to show that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a + b}$, where $a, b > 0$. 8 M

5. a) Determine the bilinear transformation which maps points $z = 1, i, -1$ on to the points $w = 2, i, -2$. 8 M

b) Show that the image of the hyperbola $x^2 - y^2 = 1$ is the lemniscate $\rho^2 = \cos 2\phi$ under the transformation $w = \frac{1}{z}$. 8 M

6. a) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. 8 M

b) State and prove the orthogonal property for Legendre's polynomials. 8 M